numbers defined by \(a_n = \frac{1}{2}(a_{n-1}^2 + 1)\) for \(n > 1\), with \(a_1 = 3\). Show that

\[
\left[ \left( \sum_{k=1}^{n} \frac{a_k}{1 + a_k} \right) \left( \sum_{k=1}^{n} \frac{1}{a_k(1 + a_k)} \right) \right]^{1/2} \leq \frac{1}{4} \left( \frac{a_1 + a_n}{\sqrt{a_1 a_n}} \right).
\]

**11443. Proposed by Eugen Ionascu, Columbus State University, Columbus, GA.** Consider a triangle \(ABC\) with circumcenter \(O\) and circumradius \(R\). Denote the distances from \(O\) to the sides \(AB, BC, CA\), respectively, by \(x, y, z\). Show that if \(ABC\) is acute then \(R^3 - (x^2 + y^2 + z^2)R = 2xyz\), and \((x^2 + y^2 + z^2)R - R^3 = 2xyz\) otherwise.

**11444. Proposed by Marian Tetiva, National College "Gheorghe Roșca Codreanu", Brașov, Romania.** Let \(k\) and \(s\) be positive integers with \(s \leq k\). Let \(f(n) = n - s\lfloor n/k \rfloor\). For \(j \geq 0\), let \(f^j\) denote the \(j\)-fold composition of \(f\), taking \(f^0\) to be the identity function. Show that

\[
\sum_{j=0}^{\infty} \left| \frac{f^j(n)}{k} \right| = -\left[ \frac{q - n}{s} \right],
\]

where \(q = \min\{k - 1, n\}\).

**11445. Proposed by H. A. ShahAli, Tehran, Iran.** Given \(a, b, c > 0\) with \(b^2 > 4ac\), let \(\lambda_n\) be a sequence of real numbers, with \(\lambda_0 > 0\) and \(c\lambda_1 > b\lambda_0\). Let \(u_0 = c\lambda_0\), \(u_1 = c\lambda_1 - b\lambda_0\), and for \(n \geq 2\) let \(u_n = a\lambda_{n-2} - b\lambda_{n-1} + c\lambda_n\). Show that if \(u_n > 0\) for all \(n \geq 0\), then \(\lambda_n > 0\) for all \(n \geq 0\).

**SOLUTIONS**

**Sums and Powers, Set Counting, and Coefficient Tracking**

**11274 [2007, 165]. Proposed by Donald Knuth, Stanford University, Stanford, CA.** Prove that for nonnegative integers \(m\) and \(n\),

\[
\sum_{k=0}^{\infty} 2^k \binom{2m+k}{m+n} = 4^m - \sum_{j=1}^{n} \binom{2m+1}{m+j}.
\]

**Solution I by Julian Hook, Indiana University, Bloomington, IN.** We show that both sides equal \(\sum_{r=0}^{2m+1} \binom{2m+1}{r}\), the number of subsets of the set \(\{1, \ldots, 2m + 1\}\) having size at least \(m + n + 1\). Such a subset may be constructed by picking an element \(z\) to be the \((m + n + 1)\)th-smallest element chosen, picking \(m + n\) elements below \(z\), and picking any subset of the elements above \(z\). With \(k = 2m + 1 - z\), the value of \(k\) indexes the choices for \(z\). Since there are \(2m - k\) elements below \(z\) from which to pick \(m + n\), and \(k\) elements above \(z\) from which to pick arbitrarily, the left side of the equation counts the specified subsets.

To count another way, note first that exactly half of the \(2 \cdot 4^m\) subsets of \(\{1, \ldots, 2m + 1\}\) have size at least \(m + 1\). From these, we eliminate those with sizes from \(m + 1\) to \(m + n\) by subtracting the sum on the right side.

© THE MATHEMATICAL ASSOCIATION OF AMERICA [Monthly 116]