Perspectives In Learning

Volume 19 | Number 1 Article 7

7-12-2021

A Mathematics Teacher's Learning Through Reflection-in-Action

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Abstract

In this study, I investigated the learning of one secondary mathematics teacher through observations in two class periods. I analyzed his learning in relation to the communities of practice framework and found reflection-in-action was instrumental in his learning. I characterized the teacher's reflection-in-action with the descriptors: developmental, hypothetical, and experimental. Developmental reflection-in-action involved the development of new understandings or practices. Hypothetical reflection-in-action involved imagined future situations. Experimental reflection-in-action involved repeated trials. I propose that the use of a cycle of reflection-in-action in professional communities may have a positive impact on teacher learning.

A teacher's first year is full of challenges, surprises, and hopefully growth. Many of us left our teacher preparation programs feeling prepared and ready to face the challenges of teaching and to focus on student learning. We believed that our personal learning would slow down as our practice matured. However, that feeling was not the case for me, the lead author. I found that my first year of teaching, and several afterward, involved a search for my teaching identity in practice that was a profound experience of growth and learning. This new experience of learning was unique and different than what I had experienced in my teacher preparation program. Instead of learning as a student in a methods course, I was learning from teaching, from my students, with my students, and for my students. Instead of having a professor orchestrating my learning experiences, I was often left on my own to make sense of the chaos. It is this common experience of teacher learning that motivated this study of how teachers learn through reflection-in-action (RiA).

Teachers around the globe experience this profoundly personal experiential learning and consider the results to be a significant source of their practice. Research on teaching and teachers has shown that teachers have specialized knowledge and practices that are unique to the profession and cultivated within the practice. From Elbaz's (1981, 1983) practical knowledge to Shulman's (1986) pedagogical content knowledge and Ball, Thames, and Phelps' et al. (2008) mathematical knowledge for teaching, we see that teachers have unique understandings that are necessary to the discipline. Theories like Schön's (1983) RiA and Schoenfeld's (2011) goal-oriented decision making have helped unpack how specialized understandings are utilized in dynamic ways within practice to address problematic situations. However, these theories do not explain how teachers learn through practice to develop specialized knowledge and practices.

As research on practice has evolved, many researchers have shifted their focus to

teacher learning and changing practices. These studies have taken many forms, but research on professional development has shown that practice must be an integral part of effective professional development experiences (Brodie & Shalem, 2011). As a result, some researchers have focused their efforts on teachers' learning through practice. For example, Margolinas, Coulange, and Bessot (2005) investigated a form of knowledge teachers develop in practice. They call it observational didactic knowledge and suggest that in-service training can play a significant role facilitating this type of practice-based learning. McDuffie (2004) showed that preservice teachers can benefit, under certain conditions, from reflections on their teaching practice. In particular, preservice teachers can learn through reflection when reflecting on teaching after the lesson, but struggle to produce any reflection while in the act of teaching. This finding suggests that Schön's (1983) RiA may be a product of experience in practice. Peterson and Williams (2008) found that the experiences of practice within student teaching and the surrounding conversations can have a profound impact on teachers' "understanding of mathematics in and for teaching" (p. 459). Graven (2004) utilized Wenger's (1998) Communities of Practice theory to investigate teacher change through professional development associated with curricular reform. She found that confidence played a central role for teachers in the process of learning and in changing practices. Horn (2005) also investigated teachers' learning in a reform effort using Wenger's (1998) theory. She found the importance of community in these teachers' changing practices and the situated nature of teachers' pedagogical understandings and practices.

All these studies about learning in practice give credibility to the importance of practice as an aspect of teacher learning. Each one provides our community with additional information about how practice intersects with and influences teacher learning. However, the wealth of research on practice and learning largely neglects the professional isolation of most teachers' day-to-day reality. Although teachers may connect with colleagues in professional relationships at various times, their practice is often unobserved by and disconnected from those communities. Some might claim that practice does not change in this reality, or at least not in a desirable manner. That stands in contrast to my own experience and research like that of Rota and Leikin (2002), who found that a beginning mathematics teacher grew in flexibility and attentiveness to students without professional development. We need to understand how change can occur in the predominant, isolated reality of mathematics teachers. Whether this change produces better learning outcomes for students or not, it is important to understand how the change occurs. Only by understanding this type of learning will we have the possibility of harnessing the majority of teachers' time for positive teacher learning.

Despite isolation from colleagues, mathematics teachers do not change in complete isolation. Teachers are members of a classroom community. My hypothesis is that changing practices occur within this community context, and the results of this study support my position. In this study, I used a social theory of learning to investigate one teacher's learning in his classroom context. Many researchers, including Graven (2004) and Horn (2005), have utilized Wenger's (1998) communities of practice theory to investigate teachers' change in the context of collegial relationships. In the absence of collegial communities of practice for the teacher in this study, I turned to his classroom community

as a potential community of practice. Although not every group of individuals can be characterized as a community of practice, I claim that this teacher's classroom communities constitute communities of practice. I support this claim in my discussion of the theoretical framework.

In the early stages of analysis of this teacher's learning I found that reflection played a significant role in his learning in practice. This finding is not surprising because reflection has been recognized as an important stimulus for teachers' learning at least since the writings of Dewey (Dewey, 1933) and has recently been included as an integral piece of how teachers integrate knowledge to build specialized understandings for teaching (Barker, Winsor, Kirwan, & Rupnow, 2020). Schön (1983) took an important step when he coined the term RiA to describe the type of in-the-moment thinking in which professionals engage. I used this RiA to analyze how this teacher learned within the context of his classroom as a community of practice because Wenger's (1998) theory does not focus on reflection explicitly as a mechanism for learning.

My initial interest in this study was the teacher's learning in classroom community. Thus, I developed my methodology around Wenger's (1998) communities of practice framework. Because a strong emerging theme of the teacher's learning was reflection, I integrated Schön's (1983) RiA as a secondary lens. For the purposes of this report I have focused my analysis around the following questions:

1. Did RiA in the context of classroom community influence this teacher's learning?
2. What forms of RiA did the teacher employ within his community of practice?

Theoretical Framework

A community of practice requires a domain, a community, and a practice (Wenger-Trayner & Wenger-Trayner, 2015). "It is the combination of these three elements that constitutes a community of practice" (p. 2). The domain defines a particular sphere of competence that helps distinguish community members from non-members. The community involves members engaging together in shared activities, assistance, and sharing information. The practice, perhaps the element that most distinctly defines a community of practice from any other community, requires the members of the community to engage in a common practice, enterprise, ongoing endeavor. All three of these elements were present in the classroom communities analyzed in this study. The domain for the teacher and his students may be defined as school mathematics in general, but more specifically as the school mathematics of class section *x*. Each class section had particular ways of approaching the mathematics and unique understandings that defined the domain of the community. These are detailed in the results section. The community consisted of the students and the teacher in each class period. Both the teacher and his students shared ideas, assisted one another, corrected one another, and participated in school mathematics together. In these communities, the practice was doing the school mathematics of class section *x*. Both the teacher and his students produced a significant amount of mathematical understandings, demonstrated mathematical practices, and performed mathematical procedures. Note that the teacher's practice in this community did not encompass the entirety of his teaching practice but was an

intersecting practice that had significant implications for his teaching practice in general. Although the teacher had a different role in relation to the domain, within the community, and as a practitioner of school mathematics than his students, he was, nonetheless, part of the community of practice. Much as communities of practice have insiders and peripheral members, this teacher held a specific role in the community while his students held differing roles. The evidence of this community of practice is elaborated in the results section through the description of the three ways that practice provides coherence to communities: mutual engagement, joint enterprise, and shared repertoire.

Wenger (1998) posited the four elements of community, practice, meaning, and identity are interwoven with learning. This paper focuses particularly on practice. Wenger suggested that practice provides coherence to communities in three ways: (a) through the mutual engagement of community members, (b) through the collective pursuit of a joint enterprise, and (c) through common tools and understandings referred to as the shared repertoire. Changes in the engagement among community members, adaptations of the joint enterprise, or expansion of the shared repertoire may indicate learning. Thus, I identified learning through the communities of practice framework. Because reflection, as a mechanism of this learning, is not addressed explicitly in Wenger's (1998) framework, I used Schön's (1983) concept of RiA to further investigate the learning identified with the communities of practice framework.

Schön's (1983) concept of RiA rests on a broad definition of reflection, for which I turned to Mewborn's (1999) synthesized aspects of reflection. Mewborn found three elements of reflection common in the literature: (a) reflection arises from a problematic situation, (b) reflection involves both thought about the situation and a resulting action, and (c) reflection is a shared experience. On the third point, Mewborn suggested that reflection "requires some outside prompting and probing," but conceded, "at least in the early stages" (p. 317). Based on this concession and the isolation of the teacher in this study, I released the third point as a requirement of reflection and instead hold it as a desired condition of reflection.

Schön (1987) described RiA as that which "serves to reshape what we are doing while we are doing it" (p. 26). Schön (1983) also noted that RiA "may stretch over minutes, hours, days, or even weeks or months, depending on the pace of activity and the situational boundaries that are characteristic of the practice" (p. 62). Thus, RiA may occur within the course of a single lesson, or it may occur as an aspect of the teacher's planning from lesson to lesson, or even year to year. The element that makes a reflection a RiA is that the one reflecting has a remaining opportunity to influence the problematic situation. Although current uses of the term RiA often exclude these cases that Schon described as occurring over extended periods of time, I returned to Schon's original formulation of the concept and allow these extended types of RiA.

Methods

I investigated a high school mathematics teacher's learning through reflection. I will call this teacher Ian (a pseudonym). Ian was in his seventh year of teaching at Buck High School (a pseudonym), a rural school in the Midwestern United States. Buck High School served a rural farming community and included 296 students in grades 9-12.

The faculty included three mathematics teachers, including Ian. In Ian's description of the school, he noted the "small school atmosphere," stating, "you know everyone on a personal level."

Data Collection

I collected data from Ian's two Math 2 classes—fourth and fifth periods. Math 2 was an integrated course that included algebra and geometry concepts intended for students in Grade 10. I collected all lesson plans and instructional materials Ian used. I observed, and videotaped, all lessons included in this study. These observations occurred over a two-week period in January immediately following winter break. I chose to observe over a concentrated time period to understand the continuity of the teacher's practice and the engagement of the community. Observations over a longer time frame but broken apart by weeks or months would afford a broader but less continuous picture of the practice and community. It would also sacrifice some depth in the understanding of the local community, although it could provide a stronger picture of the context. I asked Ian to provide reflections on his lesson planning and teaching practice. However, in my effort to capture learning without intervention, I provided minimal guidance. Thus, I did not classify any of his minimal formal reflections as reflection under the definition I use for analysis. I conducted interviews before and after the two weeks of observations. The interviews were semi-structured opportunities to address the teacher's overall change and confirm the interpretation of and reasoning for a selection of the teacher's practices. This data collection reflected my initial research interest focused on teachers' natural process of learning and not focused specifically on reflective practices.

Data Analysis

I retrospectively analyzed the data using a data reduction approach in accordance with methods outlined in Miles, Huberman, and Saldaña (2014). These methods were adapted to the communities of practice framework. I analyzed the data in four successive stages. In the first stage, I developed, checked, and utilized an initial coding scheme regarding communities of practice. In the second stage, I analyzed themes and patterns based on the first stage of coding and used the themes to describe Ian's communities of practice. In the third stage, I used the themes and patterns from stage two to describe Ian's learning in practice. Because reflection was a prominent theme in stage three, I added a fourth stage, in which I analyzed the form of Ian's reflections-in-action.

Stage one. The first stage involved the development, through emergent coding and categorizing, of a communities-of-practice coding scheme that could: (a) characterize Ian's communities of practice, and (b) illuminate the learning that occurred in practice. My coding scheme began with the three ways in which Wenger (1998) claimed practice is a source of community coherence: mutual engagement, joint enterprise, and shared repertoire. I coded lesson transcripts for evidence of mutual engagement, joint enterprise, and shared repertoire and developed subcodes for each.

During the development of the coding scheme I met with the second author to ensure that the codes were operationally well-defined in a manner consistent with the communities of practice framework, and to check for intercoder reliability. For each category (mutual engagement, joint enterprise, and shared repertoire) we check coded transcripts until we reached 85%

agreement, as suggested by Miles, Huberman, and Saldaña et al. (2014). When agreement was less than 85%, we resolved our discrepancies, adjusting operationalized definitions as necessary, and check coded additional transcripts. When we reached 85% agreement in a coding category, I used the final refined operational definitions to recode all transcripts.

Stage two. After I finished coding, I began looking for patterns and themes among the coded references. I used the capabilities of a qualitative analysis software, to help identify themes and patterns. For example, I used matrices to cross-reference coding of the content of the shared repertoire with coding of the stage of the shared repertoire. This process allowed me to see what types of content were emerging in the shared repertoire and which were well established.

Stage three. I used the coding schemes from stage one and the patterns from stage two to search for evidence of learning. I sought evidence of learning in the community from three sources: (a) in Ian's "evolving forms of mutual engagement" (Wenger, 1998, p. 95), (b) where Ian adjusted his enterprise or developed a new understanding of the joint enterprise of the community, and (c) in the initial development and adapting meanings of elements of the community's shared repertoire. In all of these areas, I sought evidence of a history of learning as well as evidence of learning that occurred during my observations. Note that I was not concerned only with learning that results in better learning outcomes for students, but any learning that occurs.

Stage four. In the fourth stage, I used the evidence of learning I found in stage three to code for situational characterizations of reflection. I labeled an instance of learning as stimulated by RiA if it involved reflection, as defined by Mewborn (1999), and matched Schön's (1987) description of RiA. Thus, an instance was labeled as RiA if it: (a) arose from a problematic situation, (b) involved both thought about the situation and resulting action, and (c) allowed the teacher the opportunity to influence the problematic situation. After labeling instances of RiA, I looked for differences in the cases, which led to three characterizations of RiA.

Results

In the first portion of this section, I focus primarily on the first three stages of analysis and Ian's learning with only brief reference to reflection. I use the communities of practice framework, first to identify instances of learning. Then, in the next portion of this section, I connect these instances of learning to Ian's reflective practices.

Identifying Instances of Learning

To demonstrate Ian's learning I share the results of my analysis of Ian's mutual engagement with his students, the joint enterprise of his classroom communities, and finally the shared repertoire of the communities. Using the Communities of Practice framework, I defined learning to be any change within the community. Two elements in these results helped reveal Ian's learning. Although, most aspects of the two classes were similar, a few elements of distinction between the classes revealed Ian's history of learning. Thus, I focused my results on those elements that revealed differences between Ian's teaching practices in the two classes. These differences provided evidence of potential change, but it is possible that any difference may be the result of other natural variation in communities. When possible, I provided additional evidence that supported my claim

of change and I further justified the claim that they are moments of learning in a later section using the concept of RiA. The second element that revealed Ian's learning was his change during the observation period. Although he did not change significantly during the two weeks, a few changes in Ian's engagement over the course of the study helped demonstrate learning through reflection.

Mutual engagement. The initial coding for mutual engagement revealed that Ian's classrooms were largely teacher-led and involved minimal interactions among students related to the mathematics content. Between 40% and 45% of mutual engagement episodes were teacher-class and between 40% and 45% were teacher-student interactions in both classes. Individual work, student-student interactions, and group work each accounted for less than five percent of mutual engagement episodes in both classes, with one exception. In Ian's fifth hour class, purposeful student-student interactions accounted for 10% of fifth hour mutual engagement episodes.

Differences in mutual engagement between the two classes. I identified several typical forms of participation that occurred in mutual engagement including questioning, mathematical telling, evaluating, and using humor (see Table 1 for definitions).

Ian used these forms of participation differently in the two classes (see Table 2). In fourth hour, the amount of questioning during teacher-class engagement was higher than the amount of mathematical telling, but the opposite was true for fifth hour. In the episodes of teacher-class engagement when Ian questioned students, fifth hour students responded more consistently and correctly than students in fourth hour. Ian received a response to his questioning in only 55% of episodes in fourth hour, but in 63% of episodes in fifth hour. Ian answered his own question in the remainder of the instances. Of the student responses, 78% were correct in fourth hour, but 92% were correct in fifth hour.

Table 1

Definitions of Typical Forms of Participation

Code	Definition
Evaluating	Instance of the teacher evaluating student work and responses for instructional purposes.
Using humor	Instance of the teacher using humor in an interaction.
Mathematical telling	Instance of the teacher making a statement that is both mathematical and instructional.
Questioning	Instance of the teacher questioning students for instructional purposes.

The amount of questioning and evaluation in teacher-student interactions was also higher in fourth hour than fifth hour. During individual work, evaluation was much more prevalent in fourth hour than in fifth hour. Furthermore, evaluation was more prevalent during fourth hour in all five types of mutual engagement. Conversely, Ian's use of humor was much more prevalent in fifth hour than during fourth hour, particularly during teacher-class and teacher-student interactions.

Differences in mutual engagement with students. In the initial interview, Ian described two different ways that he adjusted his engagement with students over time. First, he described part of his preparation for teaching in this way:

> Trying to put myself in the kids' shoes is a lot of it… If I were sitting there listening to me, am I bored, am I listening, am I understanding. Things like that. Trying to pick up, trying to catch questions, mistakes before they happen so that if a kid asks a question I can read their mind. Oh yeah, I know what you did there, you did this. Oh yeah, okay, I understand that. Things like that.

Ian used hypothetical situations to inform his teaching practice. He imagined himself as a student in order to determine how best to engage with students. Second, he described how his teaching style had developed from his motivational style of coaching. He explained,

> Each kid's motivated differently. You have to figure out what, what really gets, eats at them. You can't be too hard on some of them; they'll break so to speak. Some of them you have to be harder on. They can take that and use it positively.

Ian desired to learn about each of his students and how he could motivate them. This played out on a small scale during my observations as Ian adapted his engagement.

The following transcript excerpts illustrate the changing interactions Ian had with Emily as an example of how he adapted his engagement with students. During the first observation, Ian introduced the imaginary number *i* and did some examples simplifying expressions involving square roots. After extracting the *i* in the expression $\sqrt{-28}$ the following exchange occurred:

Ian: *i* times the square root of 28. Is that our answer?
Students: No.
Ian: No, why not? Emily, do any perfect squares go into 28?
Emily did not respond.
Ian to Emily: What are perfect squares?
Emily's response was somewhat muffled: Where one number goes into it.
Ian: Okay, so give me an example; the square root of what?
Emily: Nin...the square root of 9 is 3.
Ian: Okay, any perfect squares go into the square root of 28?
Emily: No.
Ian: What's below the square root of 9? What will give you a positive 2?
Emily: Four.
Ian: The square root of 4. Will the square root of 4 go into the square root of 28?
Emily gave no verbal response.
Another student answered Ian's next question and Ian finished simplifying the expression.

In this instance, Ian engaged Emily in an exchange he had not provided her the preparation to undertake. Thus, she hesitated on her answers, and did not respond to Ian's final question. On the second day of observations, Ian was rationalizing the denominator of a rational expression when he called on Emily again.

Ian: What's gonna happen to our denominator? Emily, what's *i* squared?
Emily: It's negative 1.
Ian: And what's the square root of 25?
Emily: 5

Later, in the same class period, Ian was rationalizing the denominator of a more complex rational expression. In order to rationalize the denominator, he needed to multiply in the expression $(-3+2i)(-3-2i)$. Ian was preparing for this step when he directed his attention to Emily.

Ian: Emily, you ready?
Emily: Yep
Ian: Deep breath, negative 3 times negative 3.
Emily: Positive 9.
Ian: Negative 3 times a negative 2*i*.

Table 2

Mutual Engagement Episodes Labeled as Mathematical Telling, Questioning, Evaluating, or Using Humor

Mutual Engagement	Mathematical telling		Questioning		Evaluating		Using Humor	
	4th	5th	4th	5th	4th	5th	4th	5th
Teacher-class	84	95	95	83	30	22	25	49
Teacher-student	19	18	56	40	36	21	14	22
Individual work	0	0	1	3	12	2	2	2
Group work	0	0	1	0	8	3	1	2
Student-student	0	0	3	7	4	2	1	2

Emily: Um, positive 6*i* square…no.
Ian: Positive 6?
Emily: *i*
Ian: Yep. Two *i* times negative 3.
Emily: Negative 6*i*.
Ian: Positive times a negative is…
Emily: Negative, 4.
Ian: Two times 2 is 4, *i* times *i* is *i* squared.

In this instance, Ian chose to engage Emily with a different type of mathematical question and walk her through the process. He provided time for Emily to prepare herself for this interaction. He asked Emily if she was ready and directed her to relax by taking a breath. Therefore, Emily was not as hesitant with her answers and she answered correctly. Although Ian did not always interact in precisely this manner, similar instances occurred in subsequent observations. This observed change in mutual engagement may be evidence of learning that was taking place within the practice of teaching, regardless of whether this change was ideal.

Joint enterprise. Differences between the enterprises of the two classes and Ian's adaptation of his plans to ensure an appropriate pursuit of what he perceived to be the community enterprise demonstrated Ian's learning in the joint enterprise. I found differences between the two classes in the accountability shared among the community members, the responsibilities taken up by the individuals pursuing the enterprise, and the negotiations about the objectives of the enterprise and how to pursue them.

Differences in the class enterprises. Ian provided several forms of accountability for his classes to pursue the enterprise of the community. The most prevalent form of accountability was the homework he assigned each day in class. However, how he held students accountable for their completion of the homework varied from class to class. For example, on the first day of observations Ian assigned the same homework to both classes, despite covering more content in fifth hour. On the following day of class with fifth hour, Ian went over the majority of the homework answers by having students share answers to the exercises. As students provided their answers, Ian kept track of exercises for which students provided incorrect answers. He assigned these exercises to pairs of students and they

wrote their method to solve the exercises on the board. Then, Ian talked about each exercise in turn until he had covered all the incorrectly answered exercises on the assignment. Ian took a different approach with fourth hour. He began the process in the same way, having students share answers to the exercises, but when an incorrect answer was provided Ian did the exercise on the board before moving on to the next exercise. He also stopped before completing all of the exercises on the assignment, saying they would return to those exercises later because he had not covered the content they needed to complete them. He returned to the remainder of the exercises the following day, but he never had the students show their work on the board as he had with fifth hour.

As I explored instances in which community members were negotiating the enterprise, I found two elements worthy of analysis. First, each instance of negotiating had a product deemed worthy of production. Second, each instance of negotiating had a community member deemed responsible for the production. The products of these instances included the completion of a mathematical task, which sometimes included an expectation for a correct answer, a mathematical explanation, a mathematical understanding, a procedural skill, a recollection of prior learning in mathematics, and a non-mathematical product. Those responsible for these products varied among the teacher, the class, or a student. Table 3 displays the frequency of each of these products and responsible parties across all of the instances of negotiation.

Two differences in the enterprises of the two classroom communities became apparent during the analysis of negotiation. The first difference involved the responsible parties. In fifth hour, individual students were held responsible for desired products in 44% of the episodes coded, but, in fourth hour, individual students were held responsible in only 30% of the episodes. In contrast, the fourth hour class as a whole was held responsible in 45% of the episodes coded, but in only 27% of episodes in fifth hour. The exception here was that students in fourth hour were held responsible 64% of the time to complete a mathematical task and the whole class was only responsible 18% of the time. In fifth hour, the responsibility to complete a task was more evenly distributed among individual students (46%) and the class as a whole (38%).

The second difference among the classes was which products were deemed important. In no instances was a mathematical understanding determined to be an important outcome for the fourth hour class. However, it was the important outcome in 22% of the episodes coded for fifth hour. In most other ways, the two classes were consistent in their desired outcomes.

The following two situations further illustrate the pursuit of mathematical understanding as part of the enterprise in fifth hour. In the first excerpt, Ian began to explain what an imaginary number is.

> Imaginary numbers, again you're gonna get more involved with this next year, they're not real numbers. George, I know what you're thinking. It's hard to go further in depth without taking other class periods to explain what it actually is. That's for next year, and for another teacher to explain. I'm just kidding, I like it when you ask those questions. Alright, so, really all you need to know: the square root of negative one, that's equal to *i*; *i* squared is equal to a negative one.

In this excerpt, Ian acknowledged that George would want to know more about imaginary numbers. Leading up to this discussion Ian had contrasted imaginary numbers with real numbers, a step he had not taken in fourth hour. After acknowledging George's desire to know more, Ian returned to his pursuit of a primarily procedural approach to the use of imaginary numbers.

In another instance, George asked Ian about who invented the quadratic formula. In this instance Ian brought the question to the attention of the whole class. He explained that he did not know who invented the quadratic formula, but took the opportunity to discuss some of the other elements he knew about the history of mathematics. He spent a significant amount of time discussing the accomplishments of Leonardo da Vinci, one of Ian's heroes.

Shared repertoire. Ian demonstrated learning in the shared repertoire in two ways. First, when the shared repertoire was different between the two classes, it showed Ian had adapted his understandings according to the community. Second, when Ian established new elements of the shared repertoire in the community, he learned new aspects of those elements for the community.

Differences in the class repertoire. Although several aspects of the shared repertoire differed between the two classes, one difference most aptly illustrates Ian's learning. After introducing the imaginary number *i* in fifth hour, Ian worked some examples simplifying expressions involving square roots. When the first instance using *i* squared came up, he called on Ben to give the value. Ben answered correctly. The next time the value of *i* squared came up, Ian called on Ben again. Every subsequent time he used the value, Ian called on Ben. This association became so prevalent in fifth hour that when Ben was absent Ian still called Ben's name to supply the value of *i* squared even though he knew he would receive no response. Then, one of the students asked if he could send Ben a text asking for the value of *i* squared. Ian did not take a similar approach in fourth hour. Ian learned this association specifically in the context of the fifth hour community. Thus, *i* took on an additional meaning for Ian in one community.

Table 3

Products of the Enterprise and Responsible Parties in Each Class

	Class		Student		Teacher		Total	
Product	4th	5th	4th	5th	4th	5th	4th	5th
Completion	2	5	7	6	2	2	11	13
Explanation	2	0	0	3	0	1	2	4
Understanding	0	4	0	6	0	3	0	13
Procedure	9	5	3	10	6	9	18	24
Recollection	4	2	0	1	0	1	4	4
Non-mathematical	1	0	2	0	2	1	5	1
Total	18	16	12	26	10	17	40	59

New elements of the shared repertoire. Students gained access to the shared repertoire of the community as they developed new mathematical conceptions and began using previously unfamiliar terminology. It was harder to make an argument that Ian was learning these elements of the shared repertoire because he was the one who introduced the majority of these elements to the community. However, I contend that Ian's use of the elements of the shared repertoire, and his attention to his students' use of the elements, went beyond simply understanding the mathematical concepts and implied that Ian was learning the shared repertoire with his students.

One way Ian demonstrated his learning about the shared repertoire was his repetition of activity. For example, his use of *i* squared in fifth hour explained above. Ian also demonstrated his learning about the shared repertoire when he recalled students' prior participations with mathematical content. For example, one student prior to my observations had referred to a level of accomplishment in a gaming world. Although it was clear that Ian did not fully understand the context of the remarks, he remembered this association with a difficult exercise and used it more than once during my observations. In one instance, Ian was solving a quadratic with complex solutions using the quadratic formula during the third lesson in fourth hour. He said, "In the past this is where we would stop. But now we're level 86, what was it, 86? At least 86 math wizards, about *i*'s and imaginary numbers and all that good stuff."

Ian recalled how individual students participated with mathematical topics in class, but he also recalled how groups of students participated in mathematical topics on assessments. For example, when he was providing an example about solving a quadratic by taking the square root of both sides of the equation he recalled, "I know this is something that many of you missed half points on quizzes, tests, so forth (pointing to $x = \sqrt{-100}$). What did I leave out, didn't do?"

Identifying Types of Reflection Influencing Learning

RiA influenced Ian's learning described above. In the following sections, I discuss why I labeled each situation RiA and how I developed three pertinent descriptors of Ian's reflections-in-action—experimental, hypothetical, and developmental. Recall that RiA meets the first two criteria of Mewborn's (1999) reflection (arising from a problematic situation, and resulting in thought and action) and Schon's (1983) description of RiA as having power to impact the problematic situation.

Experimental RiA. I refer to RiA as "experimental" whenever a process of multiple trials and adjustments is involved. Ian demonstrated the use of experimental RiA in three different ways during the course of this study. First, he adjusted his engagement with the two class periods throughout the school year. Second, he adjusted his engagement with individual students through the course of my observations. Third, he adjusted his understanding of the joint enterprise in each classroom community.

As I showed in the previous sections, Ian's engagement with fourth hour and fifth hour was not the same. I now provide an argument that the differences in his engagement were a result of learning from experimental RiA. To establish that Ian's engagement differed because of RiA I must establish the three points I used to define RiA: (a) the case arose from a problematic

situation, (b) the case involved both thought about the situation and resulting action, and (c) the case allowed Ian the opportunity to influence the problematic situation.

Ian stated that his students were not supposed to have been placed in classes based on their mathematical ability or prior achievement. He informed me that the school had eliminated all tracking so that all Grade 10 students were in Math 2 and that all Math 2 classes were intended to include a heterogeneous group of students. This perception makes it clear that Ian is not likely to have started the year with different perceptions of his classes. However, by the time of my observation, he viewed fifth hour as the "most talented class" that he ever had. His changing perception of the respective talent of each class, likely influenced his interaction so that he used more questioning and evaluating with fourth hour. He also used humor more in fifth hour, but this behavior may be less strongly associated with his perception of talent. Instead, when he discussed his use of humor with the two classes in the post-observation interview Ian remarked about fourth hour, "if we do get off task and it's, 'Okay, let's get back to math.' They struggle with that getting back on track quickly. I don't do as much [joking around]."

These differences demonstrate that Ian had a problematic situation in which two classes, which were supposed to be similar in characteristic, required, in his opinion, significantly different approaches. His comments indicated that he thought about these differences purposefully, at least in the case of his use of humor. My observations also indicated that he acted on these thoughts because he engaged differently with the two classes in terms of humor, questioning, and evaluating. The indication that Ian had changed his engagement with his classes from the beginning of the school year to the time of my observation shows that he had the opportunity to influence the problematic situation. Therefore, I classified Ian's different engagement with the two classes to be a result of RiA.

I described his RiA as experimental because it involved a process of participating in a particular way, evaluating results, and adapting participation in subsequent episodes. At the beginning of the year, I surmised that he would have had very similar interactions with the two classes. However, because fifth hour demonstrated a greater rate of answering Ian's questions and a greater percentage of correct answers, Ian decreased his questioning and evaluating over time because he felt less need to do so. This behavior would not have occurred after a single class period, but after many trials and adjustments in his engagement with each class.

The same problematic situation gave rise to Ian learning through experimental RiA in the joint enterprise. The differences in responsibility ascribed to individual students in fifth and fourth hour can be described as a result of Ian's experimental RiA. Over the course of the school year, Ian found the individual students in fifth hour more competent to meet the learning goals individually, so he gave them more responsibility for those goals. However, he felt more need to direct the learning in fourth hour, so he gave more responsibility to the class as a whole instead of to individual students. This differential treatment of fourth and fifth hour was Ian's action to alleviate the problematic situation. Thus, Ian's learning was a result of RiA. I described this situation as experimental because the changes in the joint enterprise occurred through many instances of ascribing responsibility and observing the results of that action.

Ian also used experimental RiA as he engaged with individual students. His interaction with Emily demonstrates this process. Ian's initial interaction with Emily created a problematic situation. Because she was uncomfortable in the interaction, this situation created a tension in the student-teacher relationship that could have hindered Ian's ability to teach Emily. Ian demonstrated a change in action by adjusting how he engaged with Emily. In later interactions, he prepared Emily for the questions he asked her. It is clear that he had the opportunity to influence the situation because he adjusted his actions to create better learning situations for Emily. Thus, this learning was a result of RiA. I described it as experimental because it involved multiple trials and adjustments of Ian's engagement with Emily.

Developmental RiA. As Ian learned about elements of the shared repertoire, he did so in a way that was distinct from experimental RiA. The RiA that occurred around elements of the shared repertoire resulted primarily in newly developed understandings of elements of the repertoire, as opposed to adjustments in his practice. Thus, a more accurate descriptor of these instances of RiA is developmental.

When Ian repeatedly called on a student in fifth hour to recall the value of *i* squared, he augmented his understanding of *i* squared in that community. This occurrence was not the first time that Ian had used it as an instructional technique. In the post observation interview, Ian stated, "Sometimes a certain concept or rule we have to do, I'll call on a student, just that student, every time." This practice helped him address two problems: the difficulty students have in recalling new definitions, and their difficulty remaining engaged in class discussion. Ian's action of repeatedly calling on one student helped affix the definition in students' minds because it became associated with a particular form of participation. Furthermore, Ian claimed it helped students engage in the content because they, "see that as fun," and "want to be a part of it." Thus, Ian was able to address the problematic situation by augmenting his understanding of *i* in that community.

In another instance, Ian brought up the reference to a "level 86 math wizard" in fourth hour. In this case, he had augmented his understanding of difficult mathematical tasks in that community. The problematic situation appears to involve motivating students to persevere with difficult mathematical tasks. His action of referring to this student's prior successful experience by using the same gaming reference appeared to show that he was attempting to influence the situation positively for his students. Thus, Ian augmented his understanding of difficult mathematical tasks through developmental RiA.

In these two cases, the augmented understanding of certain elements of the shared repertoire was not likely to move beyond the confines of the given classroom communities. However, other examples in the results section may have produced broader changes in Ian's mathematical knowledge for teaching. For example, his recognition that many students made a similar mistake when solving quadratic equations using square roots is useful beyond these particular classes. The problematic situation is that students commonly make this particular mistake. He took action by telling students that the mistake is common and they should attempt to avoid it. Because the problematic situation spanned beyond these individual classes, he had the opportunity to address the situation with this class and others. Thus, this developmental RiA may

have led to a broader connection with mathematical content as a part of Ian's Pedagogical Content Knowledge (PCK; Shulman, 1986).

Furthermore, the first two examples, although confined to a particular community, could also represent elements of PCK. Knowledge of particular students' strengths, weaknesses, misconceptions, and tendencies can also be an important element of mathematical knowledge for teaching, despite the limited scope that this knowledge represents. Ian called on Ben repeatedly, in part, because he knew that Ben had the correct response. In a similar way, Ian might recall a particular student's incorrect response and recognize the need to return to that student for correction. Ian's recollection of the gaming reference may be indicative of Ian's recognition of that student's continuous struggle with math and his need for success as motivation.

What Ian gained from reflecting on these particular cases may have influenced his decisions about future instructional situations. For example, when Ian first introduced *i* to fifth hour, he anticipated that George would ask probing questions about the concept because of what he gained from acquisitional RiA. Ian had his response ready so that he did not have to develop a response on the spot. He had used prior reflections to help prepare him for this particular scenario.

Hypothetical RiA. Ian also learned to engage differently with students through a process I describe as hypothetical RiA. He described this process in his initial interview in this way:

> Trying to put myself in the kids' shoes is a lot of it too. If I were sitting there listening to me, am I bored, am I listening, am I understanding. Things like that. Trying to pick up, trying to catch questions, mistakes before they happen so that if a kid asks a question I can read their mind. Oh yeah, I know what you did there, you did this. Oh yeah, okay, I understand that.

Ian described himself thinking hypothetically about how he would experience his own class as a student. In this way, he projected his own experiences as a student onto the students in his classes. The problematic situations he imagined were future oriented, but harkened back to his own prior experiences. His action regarding the problematic situation involved future engagement with students in his teaching. Because he was not reflecting purely on prior experiences, but his primary focus was on the future, I described this RiA as hypothetical.

Ian also described trying to anticipate common errors and questions. In this case, the problematic situations were the students' questions and mistakes. Because he was anticipating future occurrences, he had time to influence the problems with his future actions. Note that he did not use experience as a student here, but his experience as a teacher. He used what he learned about his students to inform his hypothetical participation. This RiA was more powerful because it involved the purposeful inclusion of understandings he developed from the very students he hoped to engage in future interactions. Thus, Ian described using his developmental RiA to inform his hypothetical RiA.

Conclusions

Ian learned through RiA in many different situations. It is clear that RiA played an important role in Ian's professional learning because of the prevalence of RiA and the variety of results. Ian used RiA to

influence his decisions about content coverage and accountability, to alter his interactions with students, and to develop new contextually based understandings of mathematical and educational concepts. He prepared for lessons and new situations through reflection.

Ian's learning through RiA can be characterized in three different ways. Experimental RiA, which involves a process of multiple trials and adjustments, occurred in both whole class and individual interactions. In hypothetical RiA, Ian relied on his past experience as a student and teacher to envision potential scenarios. He developed new understandings of the shared repertoire through his developmental reflections-in-action. However, Ian often used these different types of RiA in coordination.

Ian demonstrated that his experimental RiA influenced his developmental RiA in the case of Emily. Ian underwent an experimental process of participation with Emily as he adjusted his questioning to allow Emily to be more comfortable answering questions. Through this process he gained an understanding of Emily's interactions with mathematics. In the initial interview, Ian described trying to catch "mistakes before they happen." This initial identification of the mistake was developmental. When the mistake was identified, he used hypothetical RiA to anticipate this mistake in future situations.

There are three primary limitations of this study. First, because of the short-term nature of this study, I lacked the type of empirical evidence that Ian's engagement with both classes was the same at the beginning of the year and evolved over time. Second, Ian did not record most of his reflections explicitly so that I could observe the reflection directly. Thus, some may doubt the reality of these practices as reflection because the data do not reveal his thought process explicitly. Third, I did not evaluate Ian's learning as positive or negative. Some of the outcomes of Ian's RiA may have had a negative impact on students and the classroom environment.

Implications

This investigation of Ian's reflective practices revealed that he engaged in an extensive amount of informal reflection for teaching, despite his lack of formal reflection. I find it reasonable to assume that this may be the case for many other teachers, even if they do not appear to be explicitly reflective. Many teachers may have informal practices that may be harnessed for improved teacher learning outcomes.

In order to influence change in teachers broadly, we need to find ways to access teachers in the day-to-day reality of their practice. We cannot rely solely on large-scale, long-term professional development experiences to provide the sole means for reform across the spectrum of mathematics teachers. Instead, we need methods to help teachers access their potential for change in the reality of their isolated practice. I believe these latent reflective practices may provide the perfect opportunity to help mathematics teachers change. Just as learners of mathematics benefit from making their thinking explicit, so may teachers of mathematics. If teachers can make their reflective practices explicit, they may recognize how these practices influence their teaching. I hypothesize that through the development and honing of RiA, teachers can apply these processes purposefully to improve their instructional practice. Therefore, I propose a model of RiA cycles (see Figure 1).

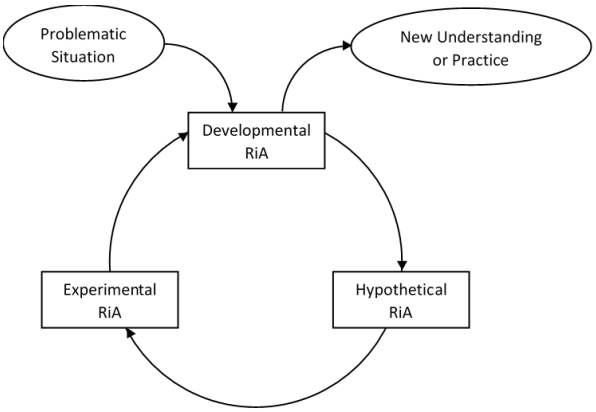

Figure 1. A Proposed Cyclical Model of Reflection-in-Action

In this cycle, a teacher first takes note of a problematic situation and the need to address the situation instructionally. Through developmental RiA, the teacher builds understanding of the problematic situation, identifying critical components of the situation, key student understandings or misconceptions, or ways in which current instruction is failing to produce the desired effects. Then, the teacher steps into hypothetical RiA to identify potential strategies to address the situation. Note that this hypothetical RiA goes beyond brainstorming solutions and into an imaginative exercise of how the intentioned strategies would play out in a hypothetical classroom scenario based on the teacher's prior knowledge and experiences. The cycle proceeds into experimental RiA when the teacher enacts the new strategy in a real situation. This experimental RiA is not merely the instructional action of the teacher, but the in-the-moment adjustment of the planned instruction. This instructional action leads to additional developmental RiA that may restart the cycle back into hypothetical RiA or result in a new instructional practice or experiential understanding of teaching and learning.

Although I did not observe some of the interactions proposed in Figure 1, they appear to have potential for promoting reflective actions that would positively influence teaching. Ian demonstrated the influence experimental reflections-in-action can have on developmental RiA, as well as the influence developmental reflections-in-action can have on hypothetical RiA. I hypothesize that hypothetical RiA may then influence experimental RiA to create this cycle of RiA as described above.

This cycle might proceed as in this example: A teacher has an unfruitful encounter with a student confused about adding and subtracting integers. While talking with the student the teacher realizes the student is having the most difficulty with subtracting negative integers—an example of developmental RiA. The teacher enters into hypothetical RiA by imagining how the student may react to an explanation of subtracting a negative using red and black chips to model the situation. Based on the hypothetical reflection, the teacher expects a positive reaction from the student and decides to use the model with the student in their afterschool meeting. Thus, the teacher introduces the new model to the student. When the student accepts the model but struggles to understand zero sum situations $(7-(-2))$ the teacher may adjust her explanation and use of the model in the moment. This behavior would be an example of experimental RiA. After the situation, she continues the cycle through developmental RiA. Note that she may proceed through the cycle several times within the afterschool session (perhaps skipping the hypothetical phase), or the cycle may continue in the days or weeks following the initial interaction.

This cycle resonates with Mewborn's (1999) definition of reflection. It begins with a problematic situation. Through the

hypothetical and experimental stages, the reflector thinks about the problematic situation and produces a resulting action to test. This cycle explicates specific forms of thought and action that might be most beneficial for teachers in the reflective process. Furthermore, it emphasizes the cyclical nature of reflective thought and action, and the desired outcome of new understandings and professional practices.

I hypothesize that if teachers are given this model and pushed to make reflections explicit, they might realize greater learning outcomes. Mewborn's (1999) third aspect of reflection could also be brought into the cycle by encouraging teachers to share their reflections. This model could be used to encourage teachers to use these types of reflections-in-action in collaborations to improve practice. The collaborative aspect might help ensure productive growth in a direction that improves student learning outcomes.

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