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11443

Author(s): Eugen Ionascu

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numbers defined by  $a_n = \frac{1}{2}(a_{n-1}^2 + 1)$  for  $n > 1$ , with  $a_1 = 3$ . Show that

$$\left[ \left( \sum_{k=1}^n \frac{a_k}{1+a_k} \right) \left( \sum_{k=1}^n \frac{1}{a_k(1+a_k)} \right) \right]^{1/2} \leq \frac{1}{4} \left( \frac{a_1 + a_n}{\sqrt{a_1 a_n}} \right).$$

**11443.** *Proposed by Eugen Ionascu, Columbus State University, Columbus, GA.* Consider a triangle  $ABC$  with circumcenter  $O$  and circumradius  $R$ . Denote the distances from  $O$  to the sides  $AB, BC, CA$ , respectively, by  $x, y, z$ . Show that if  $ABC$  is acute then  $R^3 - (x^2 + y^2 + z^2)R = 2xyz$ , and  $(x^2 + y^2 + z^2)R - R^3 = 2xyz$  otherwise.

**11444.** *Proposed by Marian Tetiva, National College "Gheorghe Roșca Codreanu", Bîrlad, Romania.* Let  $k$  and  $s$  be positive integers with  $s \leq k$ . Let  $f(n) = n - s \lfloor n/k \rfloor$ . For  $j \geq 0$ , let  $f^j$  denote the  $j$ -fold composition of  $f$ , taking  $f^0$  to be the identity function. Show that

$$\sum_{j=0}^{\infty} \left\lfloor \frac{f^j(n)}{k} \right\rfloor = - \left\lfloor \frac{q-n}{s} \right\rfloor,$$

where  $q = \min\{k-1, n\}$ .

**11445.** *Proposed by H. A. ShahAli, Tehran, Iran.* Given  $a, b, c > 0$  with  $b^2 > 4ac$ , let  $\langle \lambda_n \rangle$  be a sequence of real numbers, with  $\lambda_0 > 0$  and  $c\lambda_1 > b\lambda_0$ . Let  $u_0 = c\lambda_0$ ,  $u_1 = c\lambda_1 - b\lambda_0$ , and for  $n \geq 2$  let  $u_n = a\lambda_{n-2} - b\lambda_{n-1} + c\lambda_n$ . Show that if  $u_n > 0$  for all  $n \geq 0$ , then  $\lambda_n > 0$  for all  $n \geq 0$ .

## SOLUTIONS

### Sums and Powers, Set Counting, and Coefficient Tracking

**11274** [2007, 165]. *Proposed by Donald Knuth, Stanford University, Stanford, CA.* Prove that for nonnegative integers  $m$  and  $n$ ,

$$\sum_{k=0}^{\infty} 2^k \binom{2m-k}{m+n} = 4^m - \sum_{j=1}^n \binom{2m+1}{m+j}.$$

*Solution 1 by Julian Hook, Indiana University, Bloomington, IN.* We show that both sides equal  $\sum_{r=m+n+1}^{2m+1} \binom{2m+1}{r}$ , the number of subsets of the set  $\{1, \dots, 2m+1\}$  having size at least  $m+n+1$ . Such a subset may be constructed by picking an element  $z$  to be the  $(m+n+1)$ th-smallest element chosen, picking  $m+n$  elements below  $z$ , and picking any subset of the elements above  $z$ . With  $k = 2m+1-z$ , the value of  $k$  indexes the choices for  $z$ . Since there are  $2m-k$  elements below  $z$  from which to pick  $m+n$ , and  $k$  elements above  $z$  from which to pick arbitrarily, the left side of the equation counts the specified subsets.

To count another way, note first that exactly half of the  $2 \cdot 4^m$  subsets of  $\{1, \dots, 2m+1\}$  have size at least  $m+1$ . From these, we eliminate those with sizes from  $m+1$  to  $m+n$  by subtracting the sum on the right side.