

1985

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AUTOMORPHISMS AND RANGE FAMILIES
OF TRANSFORMATION SEMIGROUPS

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The problem of describing all automorphisms of a given semigroup of transformations of a set X has interested a number of mathematicians in the past fifty years. In 1937 Schreier [10] showed that every automorphism of the full transformation semigroup T_X is *inner* (that is, acts as a conjugation by some bijection of X). In 1952 Mal'cev [7] generalized this result by showing that every ideal of T_X has only inner automorphisms. More recently Symons [11] showed that all automorphisms of any G_X -normal semigroup (that is, invariant under a conjugation by any bijection of X) over a *finite* set X are inner, while Schein [9] produced the same result for G_X -normal semigroups of one-to-one transformations over an *infinite* set X . (See [2] for the special case of Baer-Levi semigroups.)

Chapters 2 and 3 of this thesis constitute a contribution towards the solution of the problem of describing all automorphisms of a given semigroup of transformations of an *infinite* set X . In Chapter 2 (see also [4]) we extend the well-known result from group theory, namely that any normal group of bijections of an infinite set X has only inner automorphisms, to an analogous one in semigroup theory. We show that any G_X -normal semigroup of transformations of an infinite set X has only inner automorphisms. Our purpose in Chapter 3 (see also [3]) is to offer a complete description of all automorphisms of an arbitrary *Croisot-Teissier*

Received 29 March 1985. Thesis submitted to University of Canterbury, December 1984. Degree approved March 1985. Supervisor: Dr G.R. Wood.

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\$A2.00 + 0.00.

semigroup [1], a task suggested by Schein. In this joint work with O'Meara and Wood a rich variety of automorphisms is found, ranging from inner, to "locally" inner, to thoroughly outer. We also present a description of Green's relations on Croisot-Teissier semigroups.

In Chapter 4 (and in [5]) we are concerned with the problem of a characterisation of all subsets of the power set of X , \mathcal{P}_X , which serve as sets of ranges of semigroups of transformations of X . This problem was suggested by Schein and to our knowledge has been solved only for the case of monogenic semigroups of partial transformations by Olonichev [8]. We define a *normal* subset of \mathcal{P}_X and characterise all normal subsets of \mathcal{P}_X which serve as sets of ranges of semigroups of total transformations of X . In particular, we give necessary and sufficient conditions for a subset of \mathcal{P}_X to be the set of ranges of a G_X -normal and a constant-free G_X -normal semigroup of total transformations.

In Chapter 5 (and in [6]) for a particular normal subset of \mathcal{P}_X we give necessary and sufficient conditions for an order-automorphism to be determined by a bijection of X (that is, *induced*). We then characterise those normal subsets of \mathcal{P}_X for which all order-automorphisms are induced. Apart from being of independent interest, this problem is connected with the study of automorphisms of transformation semigroups. For if an automorphism ϕ of a transformation semigroup S is inner, then ϕ produces an induced order-automorphism of the set $R(S)$ of ranges of all transformations in S . On the other hand, in instances where an automorphism ϕ of S yields an order-automorphism of $R(S)$, the knowledge that all order-automorphisms of $R(S)$ are induced can be a first step in showing that ϕ is inner.

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